Rational Functions
Rules at a Glance

$$\frac{P(x)}{Q(x)} = \frac{...px^2 + rx + s}{... + qx^2 + tx + u} = \frac{k(x-a)(x-b)...}{(x-c)(x-d)...}$$

y-intercept

- The y-intercept is the constant term in the numerator divided by the constant term in the denominator.

Zeroes

- Zeroes of the rational function are at the zeroes of the numerator.
- If the exponent on the zero is odd, then the graph transmits through the x-axis there.
- If the exponent on the zero is even, then the graph bounces off the x-axis there.
- If it is a zero in both the numerator and the denominator, then apply rules from the "points of discontinuity" below.

Vertical Asymptotes

- Zeroes in the denominator form vertical asymptotes on the graphs at the locations of those zeroes.
- If the exponent on that zero is odd, then the graph goes up on one side and comes from below on the other side of the asymptote.
- If the exponent on that zero is even, then the graph either goes up on both sides, or it goes down on both sides of that asymptote.
- If it is a zero in both the numerator and the denominator, then apply rules from the "points of discontinuity" below.
- Vertical asymptotes may not be crossed ever. They are regions of non-definition.

End-Behavior: Horizontal and Diagonal Asymptotes

- If the degree of the numerator is less than the degree of the denominator, then the horizontal asymptote is $y = 0$.
- If the degree of the numerator equals the degree of the denominator, then the horizontal asymptote is $y = k$.
- If the degree of the numerator is exactly one more than the degree of the denominator, then the diagonal asymptote is $y = kx + b$ where $kx + b$ are the first two terms of the quotient of the top polynomial divided by the bottom polynomial.
- If the degree of the numerator is 2 or more higher than the degree of the denominator, then the boundary for the end behavior is curved, we'll deal with those individually.

Points of Discontinuity

If the numerator and the denominator both have zeroes at the same x location, then it may lead to a point of discontinuity or a vertical asymptote.

- If the exponent of the top zero is more than the bottom exponent by an even number, then it is a bouncing zero with a hole at the contact.
- If the exponent of the top zero is more than the bottom exponent by an odd number, then it is an transmitting (odd) zero with a hole at the contact.
- If the exponent of the top zero is less than the bottom exponent by an even number, then it is an even vertical asymptote.
- If the exponent of the top zero is less than the bottom exponent by an odd number, then it is an odd vertical asymptote.
- If the exponent on the top zero is the same as the exponent on the bottom zero, then it is a point of discontinuity not on the x-axis.

Graph, mark and label the following rational functions completely. Make a poster out of the one you think is the coolest one.

1] $f(x) = \dfrac{2x^3 + 7x^2 + x - 10}{x^3 - 5x^2 + 7x - 3}$

2] $g(x) = \dfrac{3x^4 + 17x^3 + 21x^2 + 3x - 4}{2x^3 + 3x^2 - 23x - 12}$

3] $h(x) = \dfrac{x^5 + 11x^4 + 42x^3 + 54x^2 - 27x - 81}{x^4 + 10x^3 + 37x^2 + 60x + 36}$

4] $i(x) = \dfrac{x^4 - 5x^3 - 14x^2 + 36x + 72}{x^4 - 10x^3 + 21x^2 + 36x - 108}$

5] $j(x) = \dfrac{x^2 + 2x + 3}{x^2 + 2x - 3}$

6] $k(x) = \dfrac{x^2 + 2x - 3}{x^2 + 2x + 3}$

7] $P(x) = \dfrac{x + 4}{x - 2} + \dfrac{x - 3}{x + 1}$