How Accurate is a Spherometer in Measuring the RoC of a Telescope Mirror?

Assuming the Spherometer has legs that are the vertices of an equilateral triangle with the micrometer post dead center, the sagittal depth of the spherometer will register $R = \frac{r^2}{2\sigma}$ where R is the RoC of the paraboloidal mirror and σ is the sagittal depth.

For an example, a mirror with a diameter of 8" is measured with a spherometer whose legs are 1.5" from dead center, and the micrometer reads 0.014". So that means that without any error analysis, the RoC would be $R = \frac{1.5^2}{2*0.014} = 80.35714"$. Not only do we not know it to this many decimal places, but as a result of knowing the depth to a particular precision, that means that we know the RoC to some other particular precision as well.

Since the micrometer was read as being 0.014" that means that it was more than 0.0135" and less than 0.0145". So the error is half of 0.001", or 0.0005". This error is $d\sigma$.

If we perform a differential process to evaluate the RoC error, then we can take the derivative of both sides of $R = \frac{2.25}{2\sigma}$, or $R = \frac{1.125}{\sigma}$, which would give us $dR = 1.125(-1)\sigma^{-2}d\sigma = \frac{-1.125}{\sigma^2}d\sigma$. For the example given, that makes the error in RoC out to be $dR = \frac{-1.125}{0.014^2}*0.0005 = -2.867$". So at this point we know the RoC is between 80.4" + 2.9" and 80.4" – 2.9", or between 77.5" and 83.3". This is a huge realm of error, and that is with a micrometer with 0.001" precision. Just to mess it up even more, the radius of the feet of the spherometer were measured with something, and if they are measured with an error of plus or minus some amount, then the same error or RoC problem exists, which is yet another optimization differential calculation.

There is another way to evaluate the error, and that is to make multiple trials and do a sum of least squares RMS evaluation. If your mirror has been ground spherical and you make four measurements with the spherometer, one in the center and one at each of 120º positions about the perimeter, you can average these four readings to get your most probably RoC. Then you can determine the standard deviation, yet another σ [not the sagitta sigma]. If you go up two sigmas and down two sigma from the average, then there is a 95% chance that the actual RoC will fall between these two numbers. If you go up one sigma and down one sigma, then there is only a 68% chance of being between these two numbers, still pretty good odds.

Here is an example of that. Using the same spherometer as above, the four measurements are 0.013, 0.014, 0.015 and 0.015. Their average is 0.0145. Their squared differences are 0.00000225, 0.00000025, and a couple of 0.00000025s. Summed up is 0.000003. Divided by 4 is 0.00000075, and square rooted is 0.000866. Twice that is 0.0017. So there is a 95% chance that the sagitta is 0.0145 plus or minus 0.0017, or between 0.0128 and 0.0162. These two sagittas would give us RoCs of 69.44" and 87.89". That is a total realm of 18.45", or 78.67" plus or minus 9.23". The spherometer is extremely useless when interpreted this way. Keep in mind that I made up these data numbers. If your mirror making is really good, your four readings would have been spot-on, all 0.014 or something like that, and the RMS would be useless because the sigma would have been zero. That leaves us the differential method as the most usable method so let's improve on it.

So the ticket is to use a micrometer that has a 0.0001" readout – that would be one ten-thousandths of an inch precision markings, so the *ds* would be 5 hundred thousandths of an inch, or half a ten-thousandths of an inch., or *ds* = 0.00005" . So using that spherometer instead, let's assume that the reading was 0.0390" with 2.4898" leg radii [that's the size of my spherometer, legs based on probable center from assuming triangle was equilateral, but later you will find this quantity to be slightly not true]. That makes $R=\frac{r^2}{2\sigma}$ now equal to $R=\frac{2.4989^2}{2*0.0390}=80.0577"$, and the error for R being $dR=\frac{r^2}{2}(-1)\sigma^{-2}d\sigma=\frac{-6.21445}{2*0.039^2}*0.00005=-0.102"$. So the mirror now has an RoC of 80.06" plus or minus 0.10", or from **79.96" to 80.16".** This is very much better. We can go a step further and continue the error analysis assuming that the legs of the spherometer are also only accurately measured to the nearest 0.0005", which would approximately double my RoC error, but it is now still within 0.1" of the actual RoC.

Now for the Feet on the Spherometer.

Assuming that the feet on the spherometer are very nearly 120° apart, we can deal with how accurately the feet are from the center plunge of the micrometer, and we'll call these three distances r_1, r_2 and r_3. We can measure them to see their slight differences, average them up, and find the one that is furthest from the average. This would be or *dr*. Or, like on mine, they may all measure the same. I measured mine and they are all 2.500". Since I measured them to the nearest 0.001", then my *dr* = 0.0005". It must have been made in a very skilled machine shop.

Now we have two errors that are going to affect the RoC error, they would be *dr* and $d\sigma$. Our RoC formula is $R=\frac{r^2}{2\sigma}$, o using the Quotient Rule from differential calculus, we get

$$dR=\frac{2\sigma 2rdr-r^2 2d\sigma}{4\sigma^2}=\frac{2r\sigma dr-r^2 d\sigma}{2\sigma^2},\ \text{so}\ dR=\frac{2r\sigma dr-r^2 d\sigma}{2\sigma^2}.$$

Let's try one. Let's try an 8" mirror, Micrometer legs are 2.500" plus or minus 0.0005", reading is 0.0390", $d\sigma$ =0.00005", we get RoC = $R=\frac{2.5^2}{2*.039}=80.128"$ with an error of $dR=\frac{2*2.5*.039*0.0005-2.5^2*0.00005}{2*0.039^2}=-0.071"$. So the mirror now has an RoC of 80.12" ±0.07". That makes the RoC **80.05" to 80.19"**.

Moral of the Story: Use a spherometer with ten-thousandths of an inch precision and know your spherometer's feet radii to hopefully better than a thousandth's of an inch, hopefully a ten thousandths of an inch.