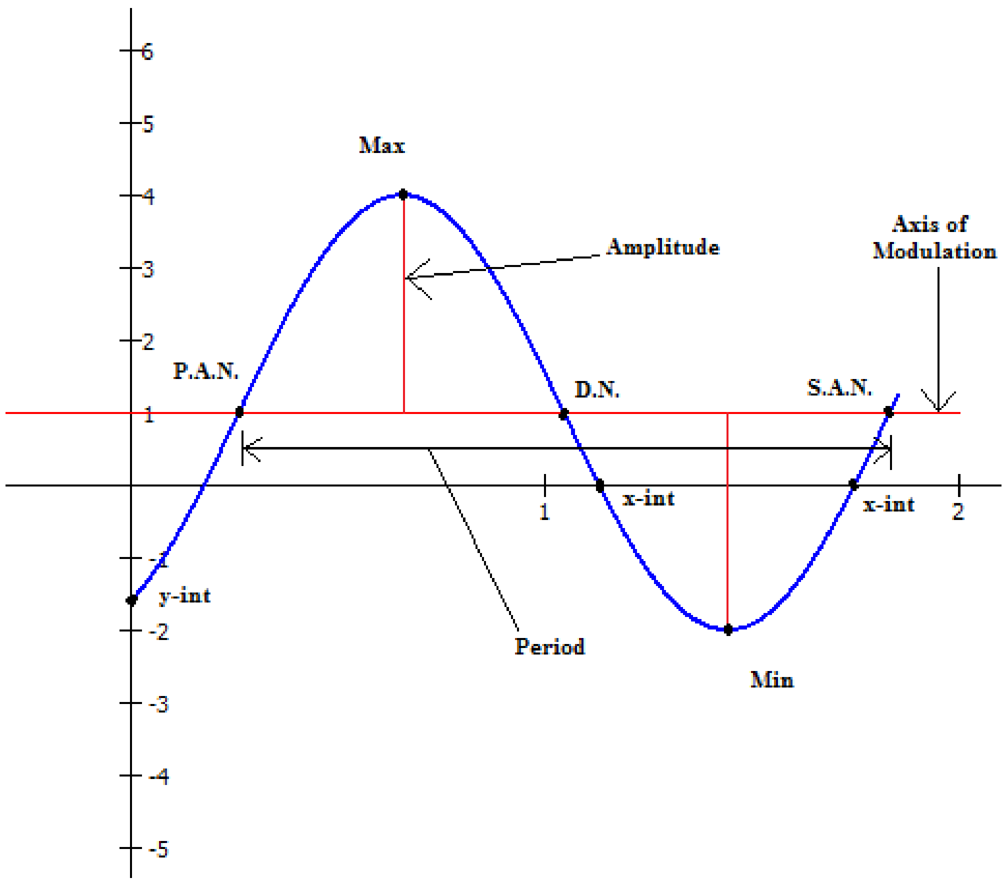
Graphing a Sinusoid

Baldwin

Consider the function $f(x) = 3\sin\left(4x - \frac{\pi}{3}\right) + 1$, which is in the form of $y = a\sin(bx + c) + d$.



1] Ask yourself "Self, when does the argument of the sine equal zero?"

$4x - \frac{\pi}{3}$ is zero when $x = \frac{\pi}{12}$, so the Principal Ascending Node [P.A.N.] is at this *x*-coordinate, and the *y*-coordinate is on the axis of modulation, or $y = 1$.

2] Ask yourself "Self, what is the period of this sinusoid?"

The period is always $2\pi/b$, so for this one it is $2\pi/4$, or $\pi/2$. This means that the Secondary Ascending Node [S.A.N.] is $\pi/2$ to the right of the P.A.N. For this particular sinusoid, that would be at $\frac{\pi}{12} + \frac{\pi}{2} = \frac{\pi}{12} + \frac{6\pi}{12} = \frac{7\pi}{12}$. The address of the S.A.N. is $(\frac{7\pi}{12}, 1)$.

3] The Descending Node [D.N.] is the midpoint of the P.A.N. and the S.A.N. So the average of the P.A.N. and the S.A.N. is $\frac{\frac{\pi}{12}+\frac{7\pi}{12}}{2}=\frac{\frac{8\pi}{12}}{2}=\frac{4\pi}{12}=\frac{\pi}{3}$, or $(\frac{\pi}{3},1)$.

4] The Maximum has an *x*-coordinate that is the average of the P.A.N. and the D.N., and a *y*-coordinate that is the Axis of Modulation [a] plus the Amplitude [d]. So $\frac{\frac{\pi}{12}+\frac{\pi}{3}}{2}=\frac{\frac{\pi}{12}+\frac{4\pi}{12}}{2}=\frac{\frac{5\pi}{12}}{2}=\frac{5\pi}{24}$, and d + a is 1 + 3 = 4, and the coordinates of the Max are $(\frac{5\pi}{24},4)$.

5] The Minimum has an x-coordinate that is the midpoint of the D.N. and the S.A.N., which is $\frac{\frac{\pi}{3}+\frac{7\pi}{12}}{2}=\frac{\frac{4\pi}{12}+\frac{7\pi}{12}}{2}=\frac{\frac{11\pi}{12}}{2}=\frac{11\pi}{24}$, and the y-coordinate of the Min is d – a = 1 – 3 = -2, and the coordinates of the Min are $(\frac{11\pi}{24},-2)$.

6] Now that you have the P.A.N., Max, D.N., Min and the S.A.N., you can now graph a smooth sinusoidal curve through these points.

7] To determine the *y*-intercept, which may or may not be in the principal period of this graph, you simply set *x* = 0 and solve for *y*. for this one that will give us

$$3\sin\left(4\times 0-\frac{\pi}{3}\right)+1=3\sin(-\frac{\pi}{3})+1=3\times\frac{-\sqrt{3}}{2}+1\approx -3\times 0.866+1=-2.598+1=-1.598$$

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8] To determine the *x*-intercepts, or the zeros, set *y* = 0 and solve for *x*.

$$0=3\sin\left(4x-\frac{\pi}{3}\right)+1\rightarrow -1=3\sin\left(4x-\frac{\pi}{3}\right)\rightarrow -\frac{1}{3}=\sin\left(4x-\frac{\pi}{3}\right)\rightarrow 4x-\frac{\pi}{3}=\sin^{-1}\frac{-1}{3}$$

$$\rightarrow -0.3398=4x-\frac{\pi}{3}\rightarrow 4x=0.707\rightarrow x=0.177$$

On this particular graph, 0.177 is an *x*-intercept, but it is not in the principal period. So we will move it to the right by 1 period, which on this problem was $\pi/2$. So 0.177 + $\pi/2$ = 1.748. That's one of them. This one is just before the S.A.N. The *x*-distance between this zero and the S.A.N. has to be equal to the distance between the D.N. and the other zero. So S.A.N. – *x*-int is $\frac{7\pi}{12}-1.748=1.833-1.748=0.085$. So to find the other one, add this amount to the D.N., which is $\frac{\pi}{3}+0.085=1.132$.

9] You're now done with the principal period. There are in fact an infinite number of *x*-intercepts. The others are intervals of the period to the right and to the left of these two. So one set are at $1.748+\frac{k\pi}{2}\ \forall k\in\mathbb{Z}$, and the other set are at $1.132+\frac{k\pi}{2}\ \forall k\in\mathbb{Z}$.

10] Now go do 30 of these so you have your skills refined. Have fun.